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ABSTRACT

The Third International Mathematics and Science Study (TIMSS) surveyed teachers and school administrators and conducted an in-depth analysis of mathematics and science curricula as well as investigating traditional student assessment of eighth grade students in 40 countries. The differences in key aspects of the educational systems in those countries suggests a cultural variation in schooling. This paper contends that the structure of mathematics and science curriculum is a reflection of culture. The structure of the data collected in TIMSS and its analyses and defining the eighth grade mathematics curriculum from the perspective of textbooks, classroom instruction, and commonalities across countries are discussed. (YDS)

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ED 458 109

The Conceptualization and Measurement of Curriculum

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Introduction

The Third International Mathematics and Science Study (TIMSS) represents the most comprehensive study of education yet undertaken. In addition to the traditionally expected student assessments, TIMSS also surveyed teachers and school administrators and conducted an in-depth analysis of mathematics and science curricula.[1] Over 40 countries participated in these various aspects of TIMSS that focused on eighth grade students.[2]

Results from the student assessments were disappointing for some countries and have prompted many policy makers to consider more carefully the curriculum portraits TIMSS produced – especially those for the highest achieving countries – in an effort to discern just what it might mean to have a "world class" mathematics curriculum.

Such an endeavor reflects one of the goals motivating any country's participation in an international comparative study like TIMSS and illustrates the value a multifaceted comparative education study can provide educators and policy makers in various participating countries as they have the opportunity to examine common practices and evaluate them from a global perspective. The goal of such studies is not to hold an international Olympics in which only one country may claim the prize but, rather, to develop a better understanding of education by which all participating countries may win.

The variation in key aspects of the education system – such as are reflected in their curricula – suggests that countries vary in their definitions of schooling and we argue that *culture* plays a significant role in these observed variations. By this hypothesis we mean that schooling itself is part of a country's culture and, therefore, what the educational beliefs or policies are – how the curriculum is organized, what constitutes teacher preparation or staff development, how the school year is parsed into days and periods – are all reflections of that culture.

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In particular we argue that what topics constitute school mathematics or school science – how topics are grouped together for study at specific grade levels, the sequence of topics studied from grade to grade, and how much emphasis each receives in any one year – are all cultural reflections. Thus, a country's curriculum is itself a cultural artifact. We conceive of the curriculum as having three instantiations: the *intended* curriculum as found in official content standards, the *potentially implemented* curriculum represented by textbooks, and the *implemented* curriculum as measured by teachers' reports of the amount of time they taught specific topics.[3] Within a given educational system, all the stakeholders have various degrees of influence in shaping these instantiations. Thus, in order to understand a given educational system, we must study the differences in these instantiations. Considered together, these three instantiations provide a triangularized portrait of the result of any one country's curricular decision making.

TIMSS collected exhaustive curriculum data such as questionnaire responses from policy makers, administrators, and school teachers, and an analyses of existing curriculum standards and textbooks used in mathematics and science classrooms. These data provide all who care about education an opportunity to study what role each instantiation plays in defining mathematics education for each country or for groups of countries who may share a common culture of mathematics education.

Educators and policy makers in different countries – especially from the point of view of this article – often hold ethnocentric viewpoints about mathematics education. These must be challenged if we are ever to gain insight from or benefit from the experiences and understandings of those belonging to systems or traditions other than our own.

Structure of the Data and Analyses

The TIMSS Mathematics and Science Frameworks provided the topic specifications used in coding the data considered here which reflect the three different curriculum instantiations: content standards, textbooks, and classroom instruction.[4] One of the fundamental goals for TIMSS was to provide a cross-national benchmark for all of the participating countries and educational systems. Motivating this goal was the desire to provide important and relevant

information for each participating country's ongoing education and curriculum reform efforts. To achieve this end it was essential that the frameworks reflect an international consensus appropriately created through collaborative development of the measurement tools – the TIMSS curriculum frameworks. In the context of a conference sponsored by the NCES in May, 1991, educators from a number of countries besides the United States spent two days analyzing and commenting on a seminal draft of the frameworks. Later that year, a revised draft was circulated for comments to all participating TIMSS representatives (SMSO, 1991). In this manner, three drafts of the frameworks were reviewed and revised through a negotiated and consensus-seeking process to produce the final published version (Robitaille, Schmidt, Raizen, McKnight, Britton, Nicol, 1993).

Representatives from each country coded their country's curricular documents, i.e., content standards and textbooks, after participating in training sessions designed and conducted by the framework's authors. In addition, the topics teachers responded to on the TIMSS Teacher Questionnaire represented the entire scope of the framework. Thirty-six of the forty-one countries that participated in the TIMSS grade eight student assessments had data for the three sources of data discussed in this article.

Since the TIMSS Mathematics Framework is organized in a hierarchy, content may be specified at several different levels.[5] Here the data has been analyzed and is discussed at two different levels. The first is at the most finely defined framework topic level of which there are 44 for mathematics. The second consists of the 20 mutually exclusive topics that were specified and covered by the TIMSS tests. Many of these are identical to the most finely defined framework topics but others are more broadly defined categories that include two or three of the 44 framework topics.

The basic structure of the data from each of these three sources can be thought of as a matrix with columns representing specific curriculum topics and rows for each TIMSS country. Depending on the specific data source, matrix elements consist of either proportions or dichotomous indicators, e.g., 0 or 1. Curriculum Content Standards were characterized either as addressing a particular framework topic (1) or not addressing a particular topic (0). For those tested topics that included more than a single framework topic, the values for the appropriate

framework topics were summed to yield the number of topics within the tested area addressed by the standard. For textbooks, each element represents the proportion of the textbook's blocks (small portions identified for coding according to well defined guidelines) that address a particular topic. Information from teachers were summarized in two ways. In one summary, each data element represents the percent of teachers within a country that addressed a specific topic in their classroom teaching.[6] In the second, each data element represents the average percent of teaching time devoted to a specific topic. The focus here, as was the case in the main TIMSS study, is on eighth grade with an eye towards developing a clear understanding of what mathematics is intended to be taught and learned according to the grade eight curriculum.

Analyses and discussion of the data follow its matrix organization and may be considered in three ways. One is to examine the differences among the row summaries (e.g., row marginals) to discover differences among countries. The second, is to look among the column summaries (e.g., column marginals) to discover variation among different topics. Finally, individual elements or cells may be examined to discover interaction effects – interesting anomalies representing unique or otherwise atypical treatment of a specific topic by a particular country. Differences in row summaries document variation between countries and differences in column summaries document variation in how individual topics are treated. But to the extent that individual elements display atypical or unique behavior, the unique flavor of mathematics within a country is documented and provides a portrait of the many different meanings that studying mathematics may have across the cultures and countries considered.

Defining the Eighth Grade Mathematics Curriculum

Content Standards

Display 1 lists the 20 assessment topics, the framework topics included in the more broadly defined test categories, as well as the framework topics not covered by the TIMSS assessment. Next to each test category is the percent of the thirty-six countries that cover the relevant framework topics as they are related to the test areas. The overall percent for a topic indicates those countries whose curriculum standards covered at least one of the framework topics represented. 'Equations and formulas', a core algebra topic, was the most widely intended

test topic as all thirty-six countries included this topic in their grade eight curriculum standards. 'Patterns, relations, and functions', another key algebra topic, exhibited similarly broad intentions as thirty-five of these thirty six countries indicated coverage (roughly 97 percent).

One of the four geometry topics, 'polygons & circles', was also intended by all but one country. The other three geometry topics, '2-D geometry', '3-D geometry & transformations', and 'congruence & similarity' were intended by 94.4 percent, 94.4 percent, and 86.1 percent respectively. Proportionality problems, a topic closely linked to equations, and 'perimeter area and volume', a topic related to geometry, round out the upper tier of the most widely intended TIMSS test topics with nearly 80 percent of the countries intending them. This suggests that internationally the most widely intended eighth grade curriculum focused on algebra and geometry topics as has been reported earlier.[7] In contrast to these most widely intended test topics are those intended by the smallest number of TIMSS countries. These topics are intended by as few as about a third, or 36 percent, of the countries – 'estimating quantity and size' – to about half, or around 50 percent, of the countries – 'estimating computations', 'rounding', and 'common fractions'.

This display creates an intriguing portrait of which TIMSS mathematics test topics were commonly intended to be studied by eighth grade students. Another interesting aspect of Display 1 is to consider the list of topics that were *not* covered on the TIMSS test but were, nonetheless, still intended by the curricula of a large number of countries. This list is important because it reflects areas that could have been tested and, perhaps, even *should* have been tested. If these widely intended but excluded topics had been included on the TIMSS test a different set of country rankings – especially for the total score – might very well have resulted. That is, certain countries' rankings could have been even lower while others' might have been higher with the inclusion of these topics on a test. So what are they?

The plausibility of the above scenario becomes more credible when one examines the actual list of topics intended by over half of the countries but not covered by the TIMSS assessment. All such topics represent more advanced mathematics including 'properties of operations', 'rational numbers and their properties', real numbers, number theory, informatics, 'vectors', and 'geometric constructions using a straightedge and compass.'

Display 1. Intended Topics by Eighth Grade Mathematics Content Standards for 36 Countries

# Tested Topic	In Standards		Not in Standards		*All* framework topics in Standards	
TIMSS Framework Topic	N	%	N	%	N	%
1 Whole Numbers	28	77.8	8	22.2	24.5	68.1
Meaning	*24*	*66.7*	*12*	*33.3*		
Operations	*25*	*69.4*	*11*	*30.6*		
2 Common Fractions	19	52.8	17	47.2		
3 Decimal Fractions & Percents	25	69.4	11	30.6	21	58.5
Decimal Fractions	*20*	*55.6*	*16*	*44.4*		
Percentages	*22*	*61.1*	*15*	*38.9*		
4 Relations of Fractions	20	55.6	16	44.4		
5 Estimating Quantity & Size	13	36.1	23	63.9	15	42.4
Estimating Quantity & Size	*13*	*36.1*	*23*	*63.9*		
Estimating Computations	*17*	*47.2*	*19*	*52.8*		
6 Rounding	18	50.0	18	50.0		
7 Estimating Computations	17	47.2	19	52.8		
8 Measurement Units	24	66.7	12	33.3		
9 Perimeter, Area, & Volume	30	83.3	6	16.7		
10 Measurement Estimations & Errors	19	52.8	17	47.2		
11 2-D Geometry	34	94.4	2	5.6	29.5	82.0
2-D Coordinate Geometry	*29*	*80.6*	*7*	*19.4*		
2-D Geometry: Basics	*30*	*83.3*	*6*	*16.7*		
12 Polygons & Circles	35	97.2	1	2.8		
13 3-D Geometry & Transformations	34	94.4	2	5.6	30.5	84.7
3-D Geometry	*31*	*86.1*	*5*	*13.9*		
Transformations	*30*	*83.3*	*6*	*16.7*		
14 Congruence & Similarity	31	86.1	5	13.9		
15 Proportionality Concepts	28	77.8	8	22.2		
16 Proportionality Problems	30	83.3	6	16.7		
17 Patterns, Relations, & Functions	35	97.2	1	2.8		
18 Equations & Formulas	36	100.0	0	0.0	36	100.0
Equations & Formulas	*36*	*100.0*	*0*	*0.0*		
Negative Numbers, Integers & Their Properties	*28*	*77.8*	*8*	*22.2*		
Exponents, Roots & Radicals	*28*	*77.8*	*8*	*22.2*		
19 Data Representation & Analysis	27	75.0	9	25.0		
20 Uncertainty & Probability	21	58.3	15	41.7		
TIMSS Framework Topics Not Tested						
Properties of Operations	*21*	*58.3*	*15*	*41.7*		
Properties of Common & Decimal Fractions	*15*	*41.7*	*21*	*58.3*		
Rational Numbers & Their Properties	*28*	*77.8*	*8*	*22.2*		
Real Numbers, Their Subsets & Properties	*25*	*69.4*	*11*	*30.6*		
Binary Arithmetic &/or Other Number Bases	*14*	*38.9*	*22*	*61.1*		
Complex Numbers & Their Properties	*8*	*22.2*	*28*	*77.8*		
Number Theory	*22*	*61.1*	*14*	*38.9*		
Systematic Counting	*12*	*33.3*	*24*	*66.7*		
Exponents & Orders of Magnitude	*13*	*36.1*	*23*	*63.9*		
Vectors	*20*	*55.6*	*16*	*44.4*		
Constructions w/ Straightedge & Compass	*25*	*69.4*	*11*	*30.6*		
Slope & Trigonometry	*15*	*41.7*	*21*	*58.3*		
Linear Interpolation & Extrapolation	*11*	*30.6*	*25*	*69.4*		
Infinite Processes	*7*	*19.4*	*29*	*80.6*		
Change	*7*	*19.4*	*29*	*80.6*		
Validation & Justification	*13*	*36.1*	*23*	*63.9*		
Structuring & Abstracting	*16*	*44.4*	*20*	*55.6*		
Informatics	*24*	*66.7*	*12*	*33.3*		

In considering the pattern of coverage for subtopics within a larger topic area, Display 1 shows that for each tested area the percent of countries that cover at least one of the subtopics as well as the percent of countries that cover each of the subtopics in that test area. For example, consider the tested area of whole numbers which contains two sub-areas – the framework topics of 'whole number meaning' and 'whole number operations.' The display shows about 80 percent of the countries cover at least one of these topics but it is also important to know what percent cover only whole number operations, only whole number meaning, and what percent cover both of these.

For this particular area, most countries that cover at least one subtopic cover them both. This generalization also holds for the other five tested areas that span multiple framework topics. In each of these cases approximately two-thirds of those countries that intend coverage of a tested area also intend it for all the subtopic areas included in the test. This makes the interpretation of the mathematics test results more straightforward since, for the majority of countries, intended curriculum coverage of a tested area also means intended coverage of all subtopics. If this were not the case, performance on an assessment area might be a reflection of which sub-areas were intended and which were not making any interpretation much less straightforward.

Display 2 indicates for each country the number of topics intended relative to both the total number of framework topics (e.g., 44) and the total number of framework topics assessed by the 20 test categories (e.g., 26). The range of the number of topics intended by content standards is quite large from a few more than 10 (Greece, Japan, and the Russian Federation) to all 44 (Iran, New Zealand, and the USA). Shifting focus to the number of possible topics covered by the TIMSS tests, the differences across countries again span from slightly less than 10 to all possible topics (in this case, 26). According to their content standards, Bulgaria, the Czech Republic, Greece, Israel, Japan, Korea, Romania, Russia, and the Slovak Republic are among those countries intending the fewest of the tested topics. Fewer countries – specifically Switzerland, Iran, Canada, Hungary, New Zealand and Norway – intended all 26 tested topics to be taught at eighth grade.

Display 2. Number of Mathematics Topics Included in the Eighth Grade Curriculum of Each Country

Country	Standards (44)	Textbook (44)	Teacher (21)	Topics Tested (26) Standards	Textbook
Australia	29	30	21	21	20
Austria	24	26	17	18	21
Belgium (Fl)	34	34	21	24	24
Belgium (Fr)	37	37	21	24	24
Bulgaria	15	14	21	11	10
Canada	35	39	21	26	26
Colombia	33	34	21	22	21
Cyprus	27	17	21	19	14
Czech Republic	15	32	21	12	23
Slovak Republic	15	32	21	12	23
Denmark	36	21	21	23	14
France	19	31	21	14	21
Germany	22	15	21	17	9
Greece	11	30	21	9	20
Hong Kong	39	23	21	24	20
Hungary	40	30	21	26	21
Iceland	30	27	21	23	20
Iran	44	22	21	26	17
Ireland	31	33	21	21	23
Israel	21	24	20	12	17
Japan	12	15	21	8	10
Korea	16	18	21	10	13
Latvia	39	44	21	25	26
Netherlands	20	28	21	14	22
New Zealand	44	27	21	26	19
Norway	32	31	21	26	22
Portugal	31	35	21	21	25
Romania	17	32	21	10	21
Russian Federation	12	19	11	9	13
Singapore	22	21	20	18	19
South Africa	21	28	21	13	19
Spain	18	27	21	14	16
Sweden	29	28	21	22	21
Switzerland	40	42	21	26	26
Scotland	36	28	21	25	23
USA	44	41	21	26	26
Slovenia	20	25	21	16	18

Assessment implications from the number of topics intended to be taught in any one year need careful consideration. For example, the Czech Republic, Korea, Japan, and the Slovak Republic all intended fewer than half of the assessed topics yet all were among the top seven performing countries on the TIMSS IRT scaled total test score. This highlights the importance of the curricular intentions for all previous grades in any substantive consideration of achievement status as it would be quite likely that, for these countries, the tested-but-not-intended topics were intended for coverage at earlier grades.[8] The specific pattern of intentions in previous grades may prove to be even more enlightening in understanding countries' performance than merely knowing whether or not topics were previously intended. That is, conclusions about an assessment given that a country had completed two years of intended emphasis on a particular topic might be quite different from the conclusions drawn given that a country introduced the topic three years previous to the assessment and still intended further instruction.

Textbooks

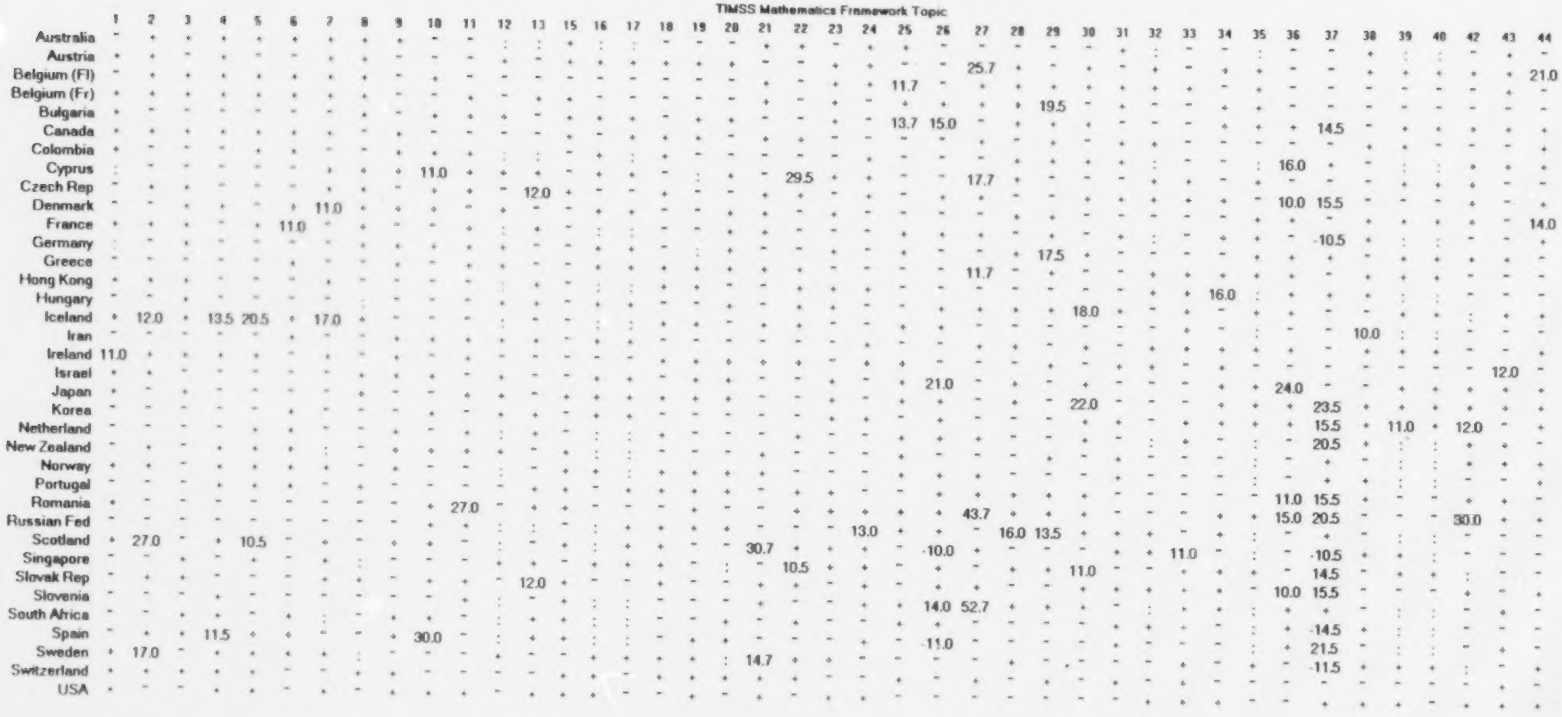
Display 3 shows the cumulative distributions for each of the 44 TIMSS framework topics in terms of the percent of the textbook that addressed that topic. Means and standard deviations are also included. The size of the standard deviation provides one indication of the size of the differences across countries in the emphasis textbooks displayed in addressing a topic. Large variations in these standard deviations likely presage *country by topic* interactions. Across all 44 topics, the standard deviations range from less than 1 to over 12 indicating that for some topics very little variation existed across countries in terms of textbook emphasis but, for other topics, variation was quite large.

Display 4 similarly shows the cumulative distribution for each country for the conditional percent of textbook emphasis typically given a topic (the conditional percent represents those statistics calculated using only the number of framework topics out of the 44 possible that the textbook actually addressed). Considering the space a textbook actually devoted to a particular topic varied across countries from a minimum of less than one percent (which accurately describes the minimum in every country) to a maximum of over 50 percent (Slovenia). The range of magnitude of the standard deviations for these conditional textbook percents in each country

Display 4. Conditional Distribution of Mathematics' Textbooks Percent Devoted to Topics

		Percentiles						
	Country	*Minimum*	*25%*	*50%*	*75%*	*Maximum*	*Mean*	*SD*
1	Australia	0.1	0.3	2.6	5.2	14.5	3.5	3.8
2	*Austria*	*0.1*	*0.1*	*0.8*	*3.5*	*28.2*	*3.9*	*7.1*
3	Belgium (Fl)	0.2	1.2	3.3	6.7	17.0	4.5	3.9
4	*Belgium (Fr)*	*0.1*	*0.8*	*2.2*	*4.7*	*22.9*	*3.8*	*4.6*
5	Bulgaria	0.2	0.3	3.2	13.3	29.8	8.2	10.1
6	*Canada*	*0.0*	*0.8*	*2.5*	*5.9*	*8.7*	*3.1*	*2.7*
7	Colombia	0.0	0.5	1.6	5.2	22.0	3.7	5.3
8	*Cyprus*	*0.4*	*1.9*	*3.5*	*10.4*	*35.2*	*7.6*	*8.9*
9	Czech Republic	0.1	1.0	2.9	9.9	32.4	6.2	7.4
10	*Slovak Republic*	*0.1*	*1.0*	*2.9*	*9.9*	*32.4*	*6.2*	*7.4*
11	Denmark	0.5	1.0	3.3	6.7	14.8	4.7	4.3
12	*France*	*0.3*	*0.7*	*2.0*	*5.3*	*16.8*	*3.3*	*3.7*
13	Germany	0.3	1.4	3.1	9.2	23.3	6.9	7.3
14	*Greece*	*0.2*	*0.4*	*1.8*	*7.7*	*14.1*	*3.9*	*4.2*
15	Hong Kong	0.1	0.8	2.6	7.2	16.4	4.6	4.8
16	*Hungary*	*0.3*	*0.6*	*1.7*	*5.0*	*18.8*	*4.0*	*5.2*
17	Iceland	0.1	0.3	2.9	11.2	21.1	5.9	6.6
18	*Iran*	*0.5*	*1.4*	*3.6*	*8.3*	*11.6*	*4.5*	*3.6*
19	Ireland	0.1	1.0	3.5	6.8	12.3	4.4	3.7
20	*Israel*	*0.1*	*0.4*	*1.3*	*4.8*	*32.4*	*5.0*	*8.8*
21	Japan	0.1	0.2	0.7	10.4	38.8	6.7	10.9
22	*Korea*	*0.4*	*1.1*	*4.8*	*10.6*	*30.8*	*6.9*	*7.5*
23	Netherlands	0.1	0.8	1.2	5.1	35.6	4.4	7.6
24	*New Zealand*	*0.1*	*1.1*	*2.3*	*6.8*	*20.2*	*3.9*	*4.3*
25	Norway	0.1	0.7	2.0	5.7	17.4	3.5	3.9
26	*Portugal*	*0.2*	*0.6*	*1.5*	*5.3*	*32.3*	*4.5*	*6.9*
27	Romania	0.2	0.7	3.0	10.0	46.9	8.4	11.9
28	*Russian Federation*	*0.3*	*0.8*	*3.6*	*15.5*	*18.9*	*6.6*	*6.8*
29	Singapore	0.2	0.8	2.2	10.4	29.8	6.1	7.4
30	*South Africa*	*0.1*	*0.3*	*1.7*	*6.6*	*14.7*	*3.8*	*4.3*
31	Spain	0.1	0.3	1.2	8.4	37.4	5.2	9.2
32	*Sweden*	*0.0*	*0.2*	*1.6*	*4.3*	*18.4*	*3.5*	*4.8*
33	Switzerland	0.0	0.5	1.5	3.9	10.3	2.7	2.9
34	*Scotland*	*0.1*	*1.2*	*3.2*	*8.2*	*33.1*	*6.1*	*7.8*
35	USA	0.0	0.8	2.2	3.8	23.6	3.1	3.9
36	*Slovenia*	*0.2*	*0.9*	*2.7*	*5.9*	*55.2*	*6.5*	*11.8*

Display 5. Median Polish of Percent of Textbook Space Devoted to TIMSS Mathematics Framework Topics



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are only somewhat smaller than the those seen in Display 3. Nonetheless, the implication concerning the magnitude of the standard deviation is similar: those countries with relatively smaller standard deviations would likely have more similar textbook space devoted to topics than those countries exhibiting larger standard deviations. Indeed, the percent of textbook space devoted to topics is not characterized by great consistency across topics. The difference between the most and least emphasized topic in countries' textbooks ranged from as little as around 10 percent (e.g., Canada and Iran) to over 50 percent (Slovenia).

As has been described, Displays 3 and 4 reveal the variability in the measures of variation for both topics across countries and for countries across topics. One way to examine these two phenomena at the same time is a median polish.[9] Display 5 gives the median polish results for the *country by topic* analysis of the percent of textbook space. This analysis was done with all 44 framework topics. Only those elements with an absolute value greater than or equal to ten are identified in the display so as to focus on differences that are likely to have a large practical impact since a deviation of 10 percent implies, on average, a 20 page difference in textbook space. An absolute difference of between five and ten percent (suggesting, on average, a 10 page difference in textbook space) is most likely also large enough to have practical impact and is noted in the table with an asterisk but our interpretive comments focus on the ten percent or greater differences.

Around two-thirds of the topics demonstrate large *country x topic* interactions in terms of textbook coverage. Using the criterion of five percent suggests major interaction effects in all but about five of the topic areas. Considering only the 26 tested topics, one finds large *country x topic* interactions in all but five of the topics – four of which are topics involving estimation and rounding while the fifth was 'proportionality concepts.' This implies that textbooks not only vary in their coverage of topics across countries but that variation also is different with respect to the specific topic being considered. Therefore, we find that each country has a unique topic-emphasis profile demonstrated by their textbook. So, for example, what ranks as the number one most-emphasized topic and by how much, on average, across countries (the marginal effect) is not necessarily the same for all countries.

In fact, the topic most emphasized as estimated from the marginals – 'equations and formulas' – also exhibits the greatest indication of *country x topic* interaction. Over one-third of all countries exhibit either larger or smaller than expected coverage for this topic. Japan, for example, has a large positive estimated interaction effect indicating more textbook space for this topic than would be predicted given their average textbook space for a typical topic and given the average amount for this topic across countries in general. Japan exhibits a similarly unusual emphasis for 'congruence and similarity.'

'Polygons and circles' and 'patterns, relations, and functions' are two other topics that exhibit a relatively large number of sizable estimated interactions effects. Textbooks as an instantiation of curriculum appear to differ appreciably across countries in terms of which topics they emphasize. As mentioned previously, another way of stating this is that textbooks differ in their content profiles – the amount of textbook space allocated to each of the 44 topics. This is consistent with our hypothesis that curriculum, itself, is a cultural artifact and, as such, varies across different cultures/countries.

In addition to the number of topics covered by each country's content standards, Display 3 also indicates the number of topics covered by countries' textbooks. With respect to all 44 TIMSS Mathematics Framework topics, this number ranges from around 15 (Bulgaria, Cyprus, Germany, Japan, and Korea), to around 40 (Canada, Switzerland, and USA) to all 44 (Latvia). With respect to those topics included on the TIMSS test, the range of topics covered in the textbooks of countries ranges from around 10 (Bulgaria, Germany, Japan, Korea, and Russia) to all tested topics (USA, Canada, Latvia, and Switzerland).

Display 4 gives the conditional means and variances for each of the countries as to the average textbook emphasis for a topic (averaged over only those topics that have some textbook coverage, i.e., topics not covered in a country textbook were excluded in determining the average). Several things from this display warrant comment. First, it is interesting to note simply the range of the number of topics found in a country's textbook. This ranges from as few as 14 (Bulgaria) to as many as 42 (Switzerland[10]). For most countries the number of topics in textbooks was in the 20's and 30's. A second observation concerns the mean percent of a textbook devoted to a topic which ranged from around three percent to as much as eight percent.

This provides one indication of the degree of focus textbooks have – the extent to which they focus on treating a select number of topics in detail as opposed to providing a brief treatment of many topics.

Finally, and perhaps even more importantly, is the demonstrated variation in the standard deviations across countries. The size of the standard deviation in the percent of textbook space devoted to topics within a country ranged from as small as about four percent to as much as around twelve percent. The larger standard deviations indicate an uneven distribution of textbook space across topics within a country and provide an indication of the degree of relative emphasis textbooks have on various topics. This implies a peak and valley focus profile as contrasted with a country having a relatively small standard deviation. In such a case, there would not be much difference in textbook space devoted to different topics resulting in a focus profile that would be quite flat. This latter profile seems to hold for the US, Norway, New Zealand, Greece, France, Canada and Australia. The more divergent focus pattern seems to hold for Slovenia, Romania, Japan and Bulgaria. In Slovenia, for example, the standard deviation is nearly 12 percentage points reflecting a range from less than one percent for one topic to as much as more than 50 percent for another! In this instance, the interquartile range was less than half the size of the standard deviation (e.g., about five percent). A similar pattern was exhibited by Japan which has a standard deviation of nearly 11 percentage points, a range from less than one percent to nearly 40 percent, and an interquartile range roughly the size of its standard deviation (e.g., about 10 percent). This contrasts sharply with the profile exhibited by the US which has a standard deviation of nearly four percent, an even smaller interquartile range of three percent, and an overall range of from less than one percent to not quite 25 percent. Similar profiles to the USA's may also be seen for Australia and Canada.(See *Many Visions, Many Aims* and *A Splintered Vision* for a fuller discussion of textbooks' focus and emphasis.)

Classroom Instruction

Under teacher coverage we consider two aspects: the percent of a country's teachers who teach a topic and the average percent of teacher time over the school year allocated to the topic. Displays 6 and 7 show the cumulative distributions for each topic in terms of the percent of teachers in a country who teach it and the average percent of teacher time spent teaching the

topic over the school year. Means and standard deviations are also included. Again, the larger standard deviations indicate topics likely to exhibit *country x topic* interactions. The standard deviations for the percent of countries' teachers teaching a particular topic ranged from a little less than ten percent (equations and formulas) to over 30 percent (data representation and analyses). This signifies much more difference across countries in terms of the percent of a country's teachers who teach data analyses as compared to the percent who teach equations and formulas. Accordingly, the range for the percent of teachers in a country who teach equations and formulas spans from slightly more than 60 percent in one country to all teachers (100 percent) in another. Similarly, the range for the percent of teachers who teach data analysis ranges from zero percent to nearly 99 percent.

Display 6. Distribution of Mean Percent of Eighth Grade Teachers Teaching Each Mathematics Topic

		Percentiles					
	topic	*Minimum*	*25%*	*50%*	*75%*	*Maximum*	*Mean*	*SD*
1	Meaning of Whole Numbers	0.0	29.6	49.4	84.9	96.6	55.4	29.0
2	*Common & Decimal Fractions*	*0.0*	*55.6*	*85.4*	*95.6*	*100.0*	*74.5*	*25.6*
3	Percentage	0.0	36.7	58.7	90.6	100.0	59.5	27.5
4	*Number Sets & Concepts*	*24.7*	*55.1*	*74.7*	*88.6*	*100.0*	*71.4*	*19.8*
5	Number Theory	0.0	24.4	42.4	76.4	95.0	48.1	28.7
6	*Estimation & Number Sense*	*6.5*	*43.5*	*63.4*	*81.2*	*96.5*	*61.1*	*24.3*
7	Measurement Units	0.0	38.2	54.0	89.4	97.7	58.0	28.9
8	*Perimeter, Area & Volume*	*27.0*	*71.4*	*84.1*	*92.7*	*100.0*	*78.2*	*19.4*
9	Measurement Estimation & Error	2.8	26.4	41.2	50.3	67.1	38.0	16.2
10	*2-D Geometry Basics*	*25.1*	*65.2*	*84.8*	*91.0*	*100.0*	*78.3*	*17.9*
11	Symmetry & Transformations	11.1	20.5	47.4	68.9	99.4	46.7	26.5
12	*Congruence & Similarity*	*11.6*	*33.5*	*64.2*	*90.9*	*100.0*	*61.4*	*29.8*
13	3-D geometry	0.0	21.8	44.9	61.0	91.0	42.1	23.5
14	*Ratio & Proportion*	*0.0*	*45.2*	*63.6*	*87.3*	*99.3*	*62.0*	*28.3*
15	Slope & Trigonometry	0.0	8.6	30.2	50.2	92.5	32.8	27.3
16	*Functions, Relations, Patterns*	*3.9*	*23.0*	*55.1*	*72.8*	*97.4*	*51.5*	*27.9*
17	Equations & Formulas	61.7	84.6	93.3	97.9	100.0	90.0	9.8
18	*Data & Statistics*	*0.0*	*17.2*	*50.3*	*77.2*	*98.5*	*48.8*	*31.1*
19	Probability & Uncertainty	0.0	4.4	8.4	27.5	66.2	18.8	21.1
20	*Sets & Logic*	*0.0*	*5.3*	*13.2*	*37.1*	*78.2*	*22.3*	*21.8*
21	Other Content	0.0	18.9	30.3	43.8	100.0	32.1	19.4

Display 7. Distribution of Mean Percent of Eighth Grade Teaching Time for Each Mathematics Topic

topic	Percentiles						
	Minimum	*25%*	*50%*	*75%*	*Maximum*	*Mean*	*SD*
1 Meaning of Whole Numbers	0.0	2.0	4.1	6.9	11.4	4.7	3.3
2 Common & Decimal Fractions	*0.0*	*5.9*	*12.5*	*18.6*	*32.2*	*12.8*	*7.6*
3 Percentage	0.0	1.7	3.8	5.6	15.7	4.2	3.3
4 Number Sets & Concepts	*1.8*	*4.0*	*6.0*	*8.5*	*18.8*	*6.8*	*3.9*
5 Number Theory	0.0	1.1	2.0	5.3	8.9	3.0	2.5
6 Estimation & Number Sense	*0.2*	*1.9*	*3.2*	*4.6*	*7.7*	*3.3*	*1.8*
7 Measurement Units	0.0	2.0	3.5	5.8	8.7	3.9	2.3
8 Perimeter, Area & Volume	*1.7*	*4.6*	*7.7*	*9.4*	*16.1*	*7.4*	*3.3*
9 Measurement Estimation & Error	0.1	1.1	1.6	2.1	3.2	1.5	0.7
10 1D & 2D Geometry Basics	*1.3*	*5.3*	*7.9*	*11.7*	*18.8*	*8.3*	*4.0*
11 Symmetry & Transformations	0.4	1.3	2.1	3.8	13.4	3.1	3.0
12 Congruence & Similarity	*0.4*	*1.5*	*3.1*	*8.5*	*23.7*	*5.5*	*5.3*
13 3D geometry	0.0	1.0	1.8	3.6	21.0	2.9	3.8
14 Ratio & Proportion	*0.0*	*2.0*	*3.5*	*5.4*	*12.0*	*3.7*	*2.6*
15 Slope & Trigonometry	0.0	0.2	1.0	2.6	14.0	2.1	3.0
16 Functions, Relations, Patterns	*0.1*	*1.1*	*3.3*	*7.5*	*14.7*	*4.8*	*4.1*
17 Equations & Formulas	4.4	8.5	13.1	17.7	24.9	13.3	5.6
18 Data & Statistics	*0.0*	*0.7*	*1.8*	*3.3*	*10.8*	*2.5*	*2.6*
19 Probability & Uncertainty	0.0	0.1	0.2	1.1	4.8	0.7	1.0
20 Sets & Logic	*0.0*	*0.1*	*0.5*	*1.5*	*7.6*	*1.3*	*1.9*
21 Other Advanced Content	0.0	1.4	2.0	2.9	18.8	2.5	3.1

Considering the percent of time devoted to teaching specific topics, the standard deviations varied from around one percent ('estimation and errors' and 'uncertainty and probability') to almost eight percent for various aspects of fractions ('common fractions', 'decimal fractions', and 'relationships of common and decimal fractions'). These topics illustrate both the great variability found in the degree of emphasis any topic within a specific country may be given from no time in one country for these fraction topics to nearly one-third of all teaching time in another as well as the lack of such variability (from virtually zero percent to about three percent for 'estimation and errors').

From Display 8 we can see that the variation across topics within a country also varies from country to country. For the percent of teachers teaching a topic, the upper limit for a topic within a country appears to be greater than 90 percent in almost all countries but the lower bound in the range varies from as low as no teachers to as high as about half of all teachers. This has an important implication for it suggests that in one country (US) every topic has at least half of its teachers covering it while in most other countries some topics have less than 10 percent of their

teachers covering it (including none or almost none) and for other topics in these same countries almost all teachers covering a different topic (e.g., more than 90 percent).

Display 9 provides the median polish results for the percent of teachers teaching a topic (a median polish for the percent of teaching time given each topic yield a similar pattern but is not displayed here). The median polish analyses clearly indicate the presence of sizable *country x topic* interactions. This is especially true for the percent of teachers (Display 9). For every topic there are numerous countries displaying at least one interaction effect greater than or equal to a ten percent difference. (This was less true for teaching time). Using the five percent difference criterion, numerous *country x topic* interactions exist. In fact, with this criterion all but two topics (neither of which are tested topics – measurement estimation and errors, and probability) exhibited interactions. Employing the ten percent difference criterion, large interaction effects are evidenced on about half of the teacher topics. For example, as noted previously for Japan in regards to textbook space, here a sizable interaction effect for 'congruence and similarity' was evidenced both for the percent of teachers teaching the topic as well as the percent of teaching time given it.

Along with the number of framework topics found in content standards and textbooks, Display 2 also indicates the average number of topics covered by teachers in each country (out of the 21 categories listed on the teacher questionnaire which represented the entire TIMSS Mathematics framework). If any teacher in a country taught a topic, that topic was considered to have been taught in that country for the purposes of Display 2. Perhaps not too surprisingly given this criterion, in virtually all countries all topics were covered by teachers. The only notable exception was Russia whose teachers taught about half of the possible topics (11 vs. 21).

Display 8. Distribution of Conditional Percent of Eighth Grade Mathematics Teachers Teaching Each Topic

	Percentiles						
	Minimum	*25%*	*50%*	*75%*	*Maximum*	*Mean*	*Std. Dev.*
Australia	10.1	47.9	72.1	90.5	95.1	66.9	24.2
Austria	*6.7*	*35.1*	*60.8*	*75.0*	*81.0*	*54.1*	*25.0*
Belgium (Fl)	1.8	14.7	44.4	69.0	87.6	44.3	27.9
Belgium (Fr)	*1.6*	*10.8*	*26.6*	*83.5*	*98.0*	*44.4*	*35.6*
Canada	8.3	49.4	82.0	93.6	99.2	69.3	28.8
Colombia	*25.5*	*37.1*	*51.9*	*70.0*	*85.7*	*54.1*	*18.9*
Cyprus	4.5	13.2	49.4	84.4	93.0	48.2	34.4
Czech Republic	*2.7*	*29.6*	*47.6*	*66.4*	*99.5*	*48.5*	*28.3*
Denmark	28.2	62.4	86.9	93.6	99.4	75.8	21.2
France	*2.6*	*29.8*	*69.0*	*85.9*	*97.0*	*59.5*	*30.1*
Germany	19.2	35.0	53.4	64.6	98.5	52.5	21.6
Greece	*4.3*	*22.7*	*40.4*	*90.2*	*94.0*	*51.6*	*33.6*
Hong Kong	2.4	17.3	42.2	77.9	87.6	45.8	30.0
Hungary	*11.7*	*61.9*	*73.0*	*85.7*	*96.4*	*70.3*	*19.8*
Iceland	4.4	14.6	43.4	85.7	100.0	46.2	35.1
Iran	*11.8*	*46.0*	*65.4*	*90.9*	*96.6*	*66.1*	*27.0*
Ireland	13.1	49.5	60.4	73.8	86.6	58.4	18.4
Israel	*4.6*	*13.4*	*26.0*	*62.0*	*94.8*	*36.6*	*27.6*
Japan	1.6	19.9	27.0	73.5	98.1	42.9	29.8
Korea	*27.8*	*43.0*	*51.4*	*79.9*	*96.8*	*58.0*	*21.4*
Latvia	6.6	22.1	39.3	56.9	97.9	42.5	25.3
Netherlands	*6.0*	*33.3*	*72.3*	*82.7*	*96.2*	*59.5*	*29.2*
New Zealand	16.1	56.6	85.1	92.3	97.9	73.2	24.2
Norway	*9.2*	*20.2*	*73.4*	*93.9*	*100.0*	*61.2*	*35.2*
Portugal	5.4	21.2	48.2	85.1	90.7	51.7	30.6
Romania	*4.5*	*26.1*	*37.5*	*50.5*	*97.1*	*42.3*	*25.9*
Russian Federation	48.4	48.6	51.6	100.0	100.0	72.7	26.1
Singapore	*0.5*	*3.0*	*81.7*	*99.2*	*100.0*	*54.5*	*47.5*
Slovak Republic	7.0	33.2	48.7	69.5	98.0	52.2	27.0
Slovenia	*4.8*	*33.5*	*48.3*	*82.5*	*100.0*	*52.3*	*29.0*
Spain	8.7	26.8	43.1	60.3	99.3	44.9	25.3
Sweden	*3.0*	*10.4*	*43.7*	*89.0*	*97.6*	*45.8*	*35.7*
Switzerland	11.3	41.1	59.9	80.0	91.0	57.2	24.3
USA	*45.8*	*58.4*	*86.5*	*92.5*	*98.2*	*76.4*	*19.1*

Display 9. Median Polish of Percent of Teachers Teaching Each Topic

	TIMSS Mathematics Teacher Topic																				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Australia	*	+	23.0	*	·	-	21.5	+	-	+	-	+	*	14.0	-	-13.8	-10.8	20.5	*	-22.0	-
Austria	-	*	16.5	*	-31.5	*	+	+	*	·	-25.5	-29.3	25.0	-59.5	-21.3	21.8	-12.3	*	*	-16.5	-27.0
Belgium (Fl)	18.1	17.0	-13.3	*	+	*	-	-18.3	-15.8	-13.8	41.8	23.0	-	*	-13.0	-32.0	·	-31.8	*	+	+
Belgium (Fr)	47.4	28.3	-14.0	35.3	45.0	17.1	-11.5	-24.0	-	-	58.0	-15.8	*	47.0	*	-29.8	24.3	-26.5	+	·	11.5
Canada	16.1	·	15.8	·	17.8	*	18.3	-	-	*	-	+	+	10.8	-37.0	-52.0	-31.0	-14.8	*	-19.3	-17.8
Colombia	28.9	*	-17.5	16.8	29.5	-11.4	·	-17.5	*	-11.0	*	-	·	-14.5	10.8	-	-	-16.0	10.3	41.5	*
Cyprus	+	17.0	30.8	·	-19.3	-	43.3	19.8	+	18.3	-24.3	-39.0	33.3	26.8	-11.0	-26.0	·	-39.8	*	-	*
Czech Rep	-17.6	-15.8	·	*	*	-19.9	-	12.0	*	-	·	47.3	17.5	10.0	33.3	51.3	18.3	-30.5	*	+	15.5
Denmark	-	-20.8	·	-10.8	·	-13.9	-	+	-	-	*	48.3	26.5	*	66.3	51.3	16.3	-22.5	-	+	37.5
France	*	*	13.5	-	+	+	18.0	-	+	-15.0	20.5	-18.3	·	-	-21.3	-28.3	-21.3	·	12.3	-10.5	*
Germany	-29.9	*	+	*	-25.3	22.9	12.3	*	-15.8	12.3	20.8	-30.0	-	12.8	57.0	*	·	14.3	-17.5	-23.3	*
Greece	-21.6	-14.8	-	-20.8	-20.0	-16.9	*	*	+	+	-	+	22.5	-	*	+	*	-15.5	+	·	11.5
Hong Kong	-19.1	-30.3	-20.5	16.8	-13.5	-28.4	-24.0	17.5	-10.0	22.0	46.5	-29.3	21.0	37.5	76.8	28.8	10.8	53.0	*	*	·
Hungary	-23.4	-	17.3	-32.5	-13.8	20.4	-	21.3	25.8	22.8	-11.8	-	*	39.3	73.5	*	*	17.8	·	-	+
Iceland	*	-11.3	-	-12.3	*	-31.4	*	-	·	·	19.5	21.8	17.0	*	-26.3	19.8	*	-	34.3	44.5	·
Iran	55.9	34.8	56.5	*	16.5	26.6	52.0	27.5	18.0	-40.0	-13.5	-22.3	-12.0	-23.5	+	-35.3	-12.3	11.0	-	-	·
Ireland	29.9	+	-12.5	+	22.5	-18.4	-12.0	-	-10.0	·	*	18.8	*	+	14.8	-55.3	*	24.0	-19.8	+	·
Israel	-	-19.8	*	-14.8	-	+	20.5	·	-	+	22.0	+	+	*	28.3	15.3	-	25.5	*	41.0	*
Japan	*	+	23.0	13.3	-	-27.9	-22.5	-21.0	-11.5	*	·	45.3	-10.5	-25.0	+	*	25.3	-11.5	14.8	64.0	*
Korea	-18.4	-36.5	-21.8	-29.5	-17.8	*	-14.3	-35.8	17.8	18.8	*	55.5	-	19.3	45.5	42.5	18.5	50.8	·	*	33.8
Latvia	*	-	-17.3	*	29.8	-20.1	-25.8	-18.3	+	+	-16.3	35.0	+	*	20.0	15.0	·	-19.8	34.5	27.8	*
Netherland	-15.4	-27.5	-20.8	*	-17.8	-23.6	+	*	11.8	20.8	-	31.5	*	-	28.5	19.5	18.5	-21.3	·	*	-
New Zealand	-	+	17.0	*	-31.0	+	10.5	*	*	*	·	-23.8	-17.5	+	-17.8	*	-16.8	19.5	+	-25.0	-30.5
Norway	21.1	·	15.8	*	16.8	*	21.3	*	+	-11.8	30.8	-28.0	*	-25.3	-28.0	-18.0	-19.0	21.3	28.5	-13.3	-
Portugal	25.4	+	23.0	+	24.0	14.1	19.5	·	+	-10.5	-40.0	-61.8	+	-58.0	-29.8	-22.8	-17.8	*	-20.3	-28.0	-20.5
Romania	-	-22.3	-14.5	14.8	46.5	10.6	·	18.5	*	21.0	13.5	40.8	-11.0	-	-10.3	14.8	+	46.0	-	*	-
Russian Fed	+	-23.8	-16.0	+	-	-21.9	*	16.0	*	-19.5	·	-	68.5	-12.0	24.3	51.3	23.3	-21.5	+	42.0	25.5
Singapore	-35.4	-60.5	-38.8	46.5	-29.8	+	-37.3	-11.8	31.8	39.8	27.3	59.5	-21.3	-44.8	41.5	11.5	27.5	-31.3	·	-	87.8
Slovak Rep	-54.9	*	30.8	18.0	-55.3	-69.1	-56.8	11.8	-44.8	12.3	47.8	32.0	-46.8	25.8	33.0	-60.0	·	40.3	-27.5	-27.3	-39.8
Slovenia	*	18.8	*	26.8	27.5	-12.4	-	-14.5	·	-13.0	-15.5	-19.3	-	*	*	22.8	18.8	·	*	+	12.0
Spain	49.9	28.8	42.5	-	*	35.6	51.0	18.5	17.0	-19.0	-21.5	-37.3	-22.0	-28.5	-12.3	-40.3	-19.3	+	+	*	·
Sweden	24.9	*	+	-17.3	24.5	11.6	28.0	-	11.0	-21.0	21.5	-16.3	·	*	-19.3	-40.3	-22.3	-27.0	*	21.5	+
Switzerland	*	-	12.5	+	24.5	+	10.0	*	·	-11.0	*	*	-11.0	12.5	-	-14.3	-23.3	+	25.3	+	*
USA	-14.1	-18.3	-	-24.3	-22.5	-13.4	-	*	·	11.0	-	35.8	54.0	40.5	12.8	22.8	13.8	-32.0	*	*	10.0

Display 8 exhibits for each country the conditional distribution over topics of the percent of teachers teaching various topics given that a topic was taught by one or more teachers in that country. Despite the rather uniform way this data appeared in Display 2, the conditional distributions provide some interesting contrasts. For example, a typical topic in the US has over three-fourths of the teachers teaching it but only about one-third of Israeli teachers teach a typical topic. However, in Israel some topics had as few as about five percent teaching it while as many as 95 percent taught another topic. The US, in contrast, still had almost half of all teachers teaching the topic that had the fewest teachers teaching it. The only other country that had this many teachers teaching the least taught topic was the Russian Federation which, you will recall, had teachers teaching only half of all the possible topics. The fact that the US had so many teachers teaching the least taught topic together with the fact that all possible topics were taught by teachers reflects the "mile wide, inch deep" nature of the US curriculum first reported in *A Splintered Vision* and further amplified in *Facing the Consequences*.

In general, most countries had between 45 and 55 percent of their teachers teaching a typical topic. Exceptions to this general trend include the Russian Federation (73 percent), Hungary (70 percent), Denmark (76 percent), Australia (69 percent), Canada (69 percent), Iran (66 percent), New Zealand (73 percent), Norway (61 percent), as well as the US and Israel as previously noted. What is interesting to note here is that even in countries with national content standards, the percent of teachers covering a typical topic does not come close to 100 percent. In the idealized case, topics not intended would not be taught by any teachers and those intended would be taught by all teachers. Given that the means in Display 8 are conditional this would result in means of 100 percent. The fact that none of the country means approach the idealized case of 100 percent implies, at a minimum, that some teachers cover topics not officially intended.

Commonalties across countries: the notion of a world core

Up to this point, we've examined for each of the three curricular instantiations some of what is typical across countries for specific topics as well as what is different. The question now arises, but what of the overlap? That is, to what degree are the three instantiations similar across

all these countries? Do the three instantiations say the same thing about what mathematics is taught at eighth grade? Is the curricular message from content standards and textbooks the same? How does the curricular message from these instantiations mesh with the classroom reality – what teachers report they actually taught?

For policymaking, these questions ultimately must be asked at the country level. Portions of these issues have already been addressed for all TIMSS countries (see especially Table 7.1 in *Many Visions, Many Aims*, p. 108) and with a particular focus on the US (see especially chapter two in *Facing the Consequences*). Here we examine these issues more directly for the whole TIMSS world using the 36 countries that provided data for all three curriculum instantiations.

Display 10 exhibits the top five topics for each of the three instantiations, including the two manifestations of teacher implementation – percent of teachers teaching and percent of teaching time. Across all these countries and according to all instantiations and measurements, the universally agreed upon number one topic for eighth grade mathematics – the one most intended by content standards, most covered by textbooks, taught by the most teachers, and given the most teaching time –was equations and formulas. Also in the top five across all instantiations – the top three, actually – was 2-D geometry (most likely 'polygons and circles' as the teacher category listed also includes two additional geometry topics).

On the policy side, functions and 3D geometry are among the top five for both content standards and textbooks yet neither appear among the top five implemented topics. Topics not included in the policies of most countries but actually taught are topics related to fractions and number theory. 'Perimeter, area and volume' appears in the top five for all but the content standards where it actually is the sixth most intended topic. One might conjecture that the inclusion of fractions and number theory at the level of implementation but not at the level of prescription (policy) reflects teachers' beliefs about the necessity of covering a topic prescribed at earlier grades but not yet mastered by students.

Display 10. Top Five Topics for Each Instantiation

Content Standards	Textbooks
1. Equations & Formulas	1. Equations & Formulas
2. Patterns, Relations & Functions	2. 2-D Geometry: Polygons & Circles
3. 2-D Geometry: Polygons & Circles	3. Patterns, Relations & Functions
4. Congruence & Similarity	4. 3-D Geometry
5. 3-D Geometry	5. Perimeter, Area & Volume

% of Teachers	% of Instructional Time
1. Equations & Formulas	1. Equations & Formulas
2. 1D & 2D Geometry Basics	2. Common & Decimal Fractions
3. Perimeter, Area & Volume	3. 1D & 2D Geometry Basics
4. Common & Decimal Fractions	4. Perimeter, Area & Volume
5. Number Sets & Concepts	5. Number Sets & Concepts

Conclusions

From these descriptions of the three curricular instantiations, two conclusions are warranted. The first is that it is quite clear that there are different cultural approaches, as exhibited by the differences across countries, in the way mathematics is defined for eighth grade students. These different cultural approaches are manifested by each of the three instantiations of curricula: content standards, textbooks, and teachers' instruction. It is clear that there is more than one way to do eighth grade mathematics. Looking at the specific topics included and emphasized both across the three instantiations within each country and comparing these across countries, the mathematics students study in one country can look quite different from what students in another country study. These cultural differences in what constitutes eighth grade mathematics are quite likely not inconsequential – not only for how students might perform on any given assessment but for their future learning as well. Even if all the different cultural approaches to mathematics schooling yielded identical performances at the eighth grade level, one might still find that the particular curricular pattern observed in one country provided a firmer foundation for advanced mathematics than other alternatives. Given the various performance patterns observed on the TIMSS eighth grade and end of secondary mathematics tests, both of these ideas appear quite plausible. Currently we are pursuing more formal analyses

relating the aspects of curricula presented here to various country level measures of students' performance.

The second conclusion is that even though countries may strive to have alignment across the three curricular instantiations, there remains some variation in the definition of what constitutes eighth grade mathematics within any one country. The fact that the way in which these three instantiations vary amongst themselves from one country to the next simply reinforces the first conclusion. That is, there is no single way in which the three instantiations differ from one another from one country to the next. This fact emphasizes the cultural context in which these instantiations have been developed and operate. Furthermore, it accentuates the folly of adopting in a wholesale fashion the curricular patterns observed in an alien culture. Clearly we can (must?) learn from other cultures but these lessons must be thoughtfully analyzed and creatively translated into our own unique cultural context for education. Failing to recognize the cultural nature of schooling and measures of it precludes useful insights and conclusions being developed for improving educational policies and practice.

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[1] *Pursuing Excellence: A Study of U.S. Eighth-Grade Mathematics and Science Teaching, Learning, Curriculum, and Achievement in International Context*. NCES 97-198 (U.S. Government Printing Office, Washington, D.C. 1997).

Pursuing Excellence: A Study of U.S. Fourth-Grade Mathematics and Science Achievement in International Context. NCES 97-255 (U.S. Government Printing Office, Washington, D.C. 1997).

Pursuing Excellence: A Study of U.S. Twelfth-Grades Mathematics and Science Achievement in International Context. NCES 98-049 (U.S. Government Printing Office, Washington, D.C. 1998).

[2] The TIMSS defined three student populations: the two adjacent grades containing the majority of nine-year-olds (Population 1), the two adjacent grades containing the majority of thirteen-year-olds (Population 2), and students in the final year of secondary school (Population 3). The focus here is on population 2 students in the upper grade. Although the name of this grade level varied from country to country, with few exceptions these students had had eight years of formal schooling hence, here, we use 'eighth grade' to refer to these students.

[3] Schmidt, W. H., Jorde, D., Cogan, L. S., Barrier, E., Gonzalo, I., Moser, U., Shimizu, K., Sawada, T., Valverde, G., McKnight, C., Prawat, R., Wiley, D. E., Raizen, S., Britton, E. D., & Wolfe, R. G. (1996). *Characterizing Pedagogical Flow: An Investigation of Mathematics and Science Teaching in Six Countries*. Dordrecht/Boston/London: Kluwer.

[4] Initially, the frameworks were presented and explained only in technical reports but have been reproduced, documented and published. See Robitaille, D. F., Schmidt, W. H., Raizen, S., McKnight, C., Britton, E., & Nicol, C. (1993). *Curriculum Frameworks for Mathematics and Science* (TIMSS Monograph No. 1). Vancouver: Pacific Educational Press.

[5] For further discussion of the TIMSS frameworks as they relate to coding and analyzing documents, see Schmidt, et al, 1996, or one of the international curriculum analysis volumes: Schmidt, W. H., McKnight, C., Valverde, G. A., Houang, R. T., & Wiley, D. E. (1997). *Many Visions, Many Aims, Volume I: A Cross-National Investigation of Curricular Intentions in School Mathematics*. Dordrecht/Boston/London: Kluwer. Or Schmidt, W. H., Raizen, S. A.,

Britton, E. D., Bianchi, L. J., & Wolfe, R. G. (1997). *Many Visions, Many Aims, Volume II: A Cross-National Investigation of Curricular Intentions in School Science*. Dordrecht/Boston/London: Kluwer.

[6] The TIMSS sampling design was based on the number of students in each grade. A stratified random sample of schools was selected with probability proportional to the number of students in the sampled grade. Mathematics classrooms were then randomly selected from each school and these students' teachers responded to the TIMSS Teacher Questionnaire. Responses from Teacher questionnaires were weighted by the number of students each teacher represented. Therefore, reference to the percent of teachers in a country refers to the percent of students who had such teachers and reference to instructional time refers to the amount of instructional time students experienced averaged across all students.

[7] Schmidt, W. H., McKnight, C., Cogan, L. S., Jakwerth, P. M., & Houang, R. T. (1999). *Facing the Consequences: Using TIMSS for a Closer Look at US Mathematics and Science Education*. Dordrecht/Boston/London: Kluwer.

Schmidt, W. H., McKnight, C., & Raizen, S. (1997). *A Splintered Vision: An Investigation of U.S. Science and Mathematics Education*. Dordrecht/Boston/London: Kluwer.

Valverde, G. A., & Schmidt, W. H. (in press). Greater Expectations. *Journal of Curriculum Studies*.

[8] This is why we have focused on gain at the subtest level in other publications that discuss the assessment results. As a brief example, we might expect small gains in the 18 framework topics Japan does not intend to be taught at eighth grade. The overall gain score would not likely be sensitive to this type of differential gain across topics. In addition, the overall status measure (i.e., the single IRT derived score) would similarly be unlikely to demonstrate any effect of differential topic intentions. For some countries this may raise the question of how fair the test is if some tested topics are not intended to be covered by them until a later grade or not intended at all. See our *Facing the Consequences* for a fuller discussion of these issues.

[9] A median polish examines the contents of a two-way table (or data matrix) repeatedly removing row and column effects from each element. Once the marginals approach zero, cells exhibiting relatively larger absolute values indicate sites of interaction between the row and column factors. As employed here, for example, median polishing thus identifies unusual values (i.e., *topic x country* interactions) by comparing typical (median) values for a topic across all TIMSS countries and each country's typical (median) value across all topics. For more, see Hoaglin, D. C., Mosteller, F., and Tukey, J. W. (eds.). 1985. *Exploring Data Tables, Trends, and Shapes*. New York: John Wiley & Sons.

[10] Switzerland has three distinct education systems reflecting three different primary languages: German, French, and Italian. Each of these have their own content standards and textbooks. Data in the displays for Switzerland represent an aggregate over the three systems which provides an appropriate indicator for the country as a whole but would not be applicable to any particular student.